Technical Memorandum

AN ANALYTICAL METHOD
FOR THE DESIGN
OF SCORED RUPTURE DIAPHRAGMS
FOR USE IN SHOCK AND GUN TUNNELS

by R. W. HENDERSON

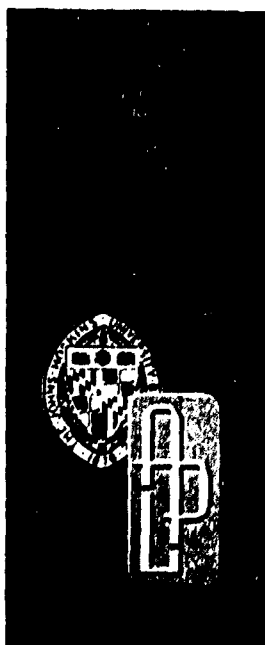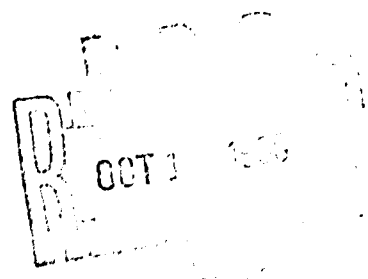TG–902
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SILVER SPRING MARYLAND

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LIST OF SYMBOLS

A	cross-sectional area after straining
A_o	cross-sectional area before straining
F	force normal to cross-sectional area
F_p	force normal to pressurized surface
P	pressure
S	true stress
S'	nominal stress
S_u	ultimate stress
a	radius of rounded edge of opening
b	radius of opening
e	base of natural logarithms
ℓ	length after straining
ℓ_o	length before straining
m	thinning coefficient (defined in text)
r_1	inside radius of curvature of strained diaphragm
r_2	radius of curvature at groove bottom
r_3	outside radius of curvature of strained diaphragm
t	thickness of diaphragm material
α	half angle included by radii at opening
δ	true strain
ϵ	nominal strain

Subscripts

g	at grooved portion of diaphragm
b	at burst condition

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AN ANALYTICAL METHOD FOR THE DESIGN OF SCORED RUPTURE DIAPHRAGMS

FOR USE IN SHOCK AND GUN TUNNELS

R.W. Henderson

ABSTRACT

A method of determining burst pressure and petal curvature
for a particular diaphragm opening, thickness and score depth
is presented which utilizes the plastic stress-strain relation-
ship for the diaphragm material as determined by a conventional
tensile test. A comparison of experimental and predicted
results is included.

INTRODUCTION

Scored, metal diaphragms are used extensively in shock tubes and
gas guns as quick-opening devices. Design of these rupture diaphragms has
generally relied on empirical or semi-empirical methods.[1] The practice of
scoring so that the diaphragm will open in discrete sections or petals is
fairly universal. Two score marks or grooves are usually cut at right
angles in one face. Where the depth of the groove is an appreciable portion
of the diaphragm thickness, its influence on the burst pressure is evident
and must be taken into account. In operation the diaphragm bulges outward
into the open area opposite the side being pressurized, the bulge
resembling a portion of a sphere. At burst the diaphragm breaks along
the score marks into four petals which fold back out of the main flow
stream against the sides of the opening.

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Although scoring helps it does not necessarily eliminate

undesirable behavior at rupture, such as incomplete opening and breaking

off of parts of the diaphragm. In the case of APL's Hypersonic Gun

Tunnel,[2] many of the original diaphragms had one or more of their

petals knocked off (or bent back across the opening) by the shock waves

which are reflected within the gun compression unit. The pieces of

flying metal so produced constituted a hazard to the gun's smooth precision

bore. The design of these diaphragms was similar to ones successfully

used with shock tunnels at the Naval Ordnance Laboratory, White Oak,

Maryland. The material used in each instance was 304 stainless steel.

Design thicknesses for the series of burst pressures desired for operation

at APL were determined from hydraulic burst test of a number of samples of

varying thickness. The diaphragms for the higher burst pressures (and in

many instances the intermediate burst pressures) in this original series

had one or more of their petals knocked off or bent back across the opening

when used in the gun. The petals of these diaphragms appeared to have a smaller

radius of curvature, that is, to have bulged further into the confine of the

opening before rupturing, than those diaphragms which retained their petals

(or did not have them bent back across the opening). This led to the

speculation that the radius of curvature of the bulge was a factor in petal

behavior. With a short radius of curvature the tips of the petals after

rupture would project further into the flow stream, whereas with a large

radius of curvature they would lie relatively flat against the walls of

the opening.

The bending of the petals laterally by the action of the
gas following rupture makes for considerable uncertainty in measuring
the amount of curvature at the insta.t of burst. However, examination of
specimens of the design ruptured by hydraulic pressure seemed to confirm
a difference in curvature between diaphragms having different design burst
pressures. Since the only design variables for a particular burst pressure,
material and size of opening are the diaphragm thickness and groove
depth, the question is how, if at all, do they affect curvature. The
further observation that those diaphragms which lost petals (hence those
with apparently smaller radius of curvature) had larger groove to panel
thickness ratios indicated the possibility of a relationship of this
parameter to curvature.

The following analysis shows that a relationship between thickness
ratio and curvature does exist and that burst pressure and petal curvature
are functions of the two thicknesses - burst pressure being primarily a
function of thickness at the groove bottom, and curvature being a function
of diaphragm thickness. The method used is considered to be generally
applicable to the design of flat, scored, rupture-diaphragms and may be
employed to control petal curvature as well as to determine burst pressure
on the basis of diaphragm thickness and score depth.

ANALYSIS

For the purpose of this analysis a diaphragm of the design

shown in Fig. 1a is used. Pressure is applied on the grooved side

with the opposite side facing a circular opening having a rounded edge,

Fig. 1b. Under the action of pressure, part of the diaphragm bulges

outward into the opening, Fig. 2. Treating this part of the diaphragm

as a portion of a thin-walled sphere with an inside radius, r_1, the

stress in the wall may be determined by considering an element of the

wall with four of its faces under tension of a force, F, Fig. 2. A

force, F_p, due to the pressure, P, acts on the inside face of the element

over an area $(\Delta \ell)^2$ and

$$F_p = P(\Delta \ell)^2 = P \; r_1 \Delta \alpha \; \Delta \ell. \tag{1}$$

This force is balanced by the four tensile forces acting on the lateral

faces of the element. From a vector diagram, Fig. 2, of the forces in

one plane,

$$F \Delta \alpha = 1/2 \; F_p. \tag{2}$$

Substituting for F_p

$$F = 1/2 \; P \; r_1 \; \Delta \ell. \tag{3}$$

The area of each lateral face is

$$A = t/m \; \Delta \ell \tag{4}$$

where t is the initial thickness and m is a coefficient to correct

for the thinning caused by stretching the material. Combining terms, the

following expression for the stress, S, in the strained diaphragm wall

is obtained:

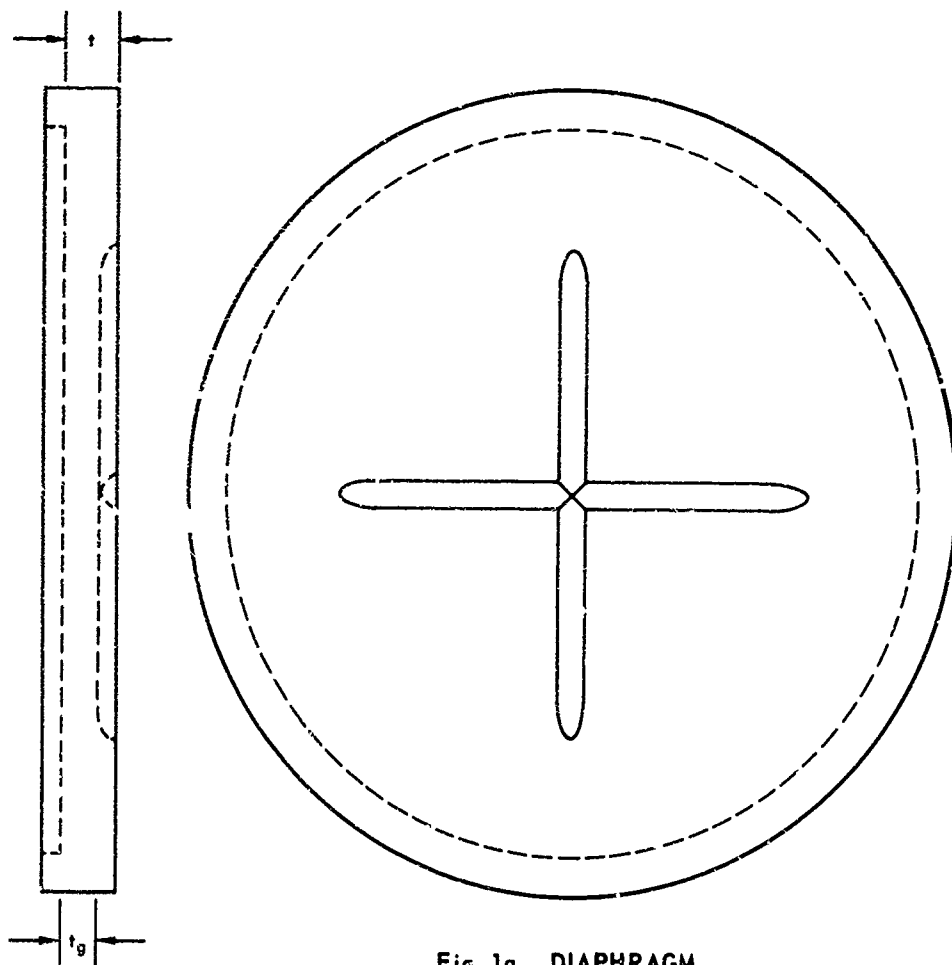$$S = \frac{F}{A} = \frac{\frac{1}{2} \; Pr_1 \; \Delta \ell}{\frac{t}{m} \; \Delta \ell} = \frac{Pr_1}{2 \frac{t}{m}}. \tag{5}$$
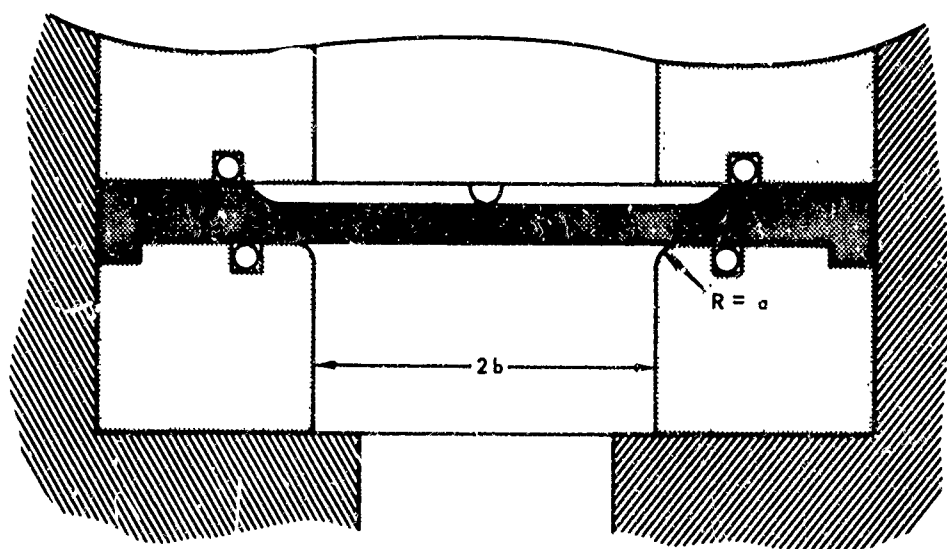
Fig. 1a **DIAPHRAGM**
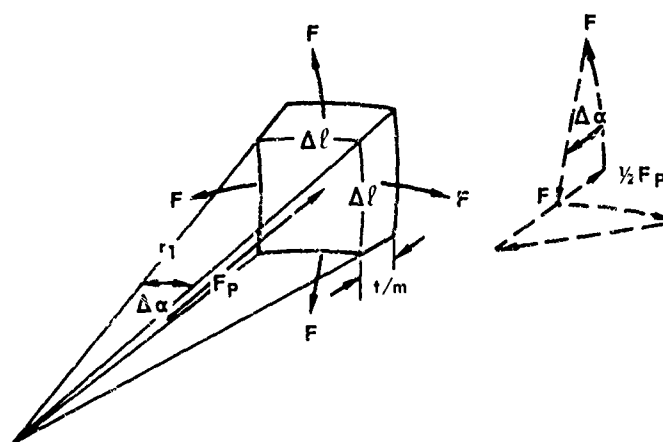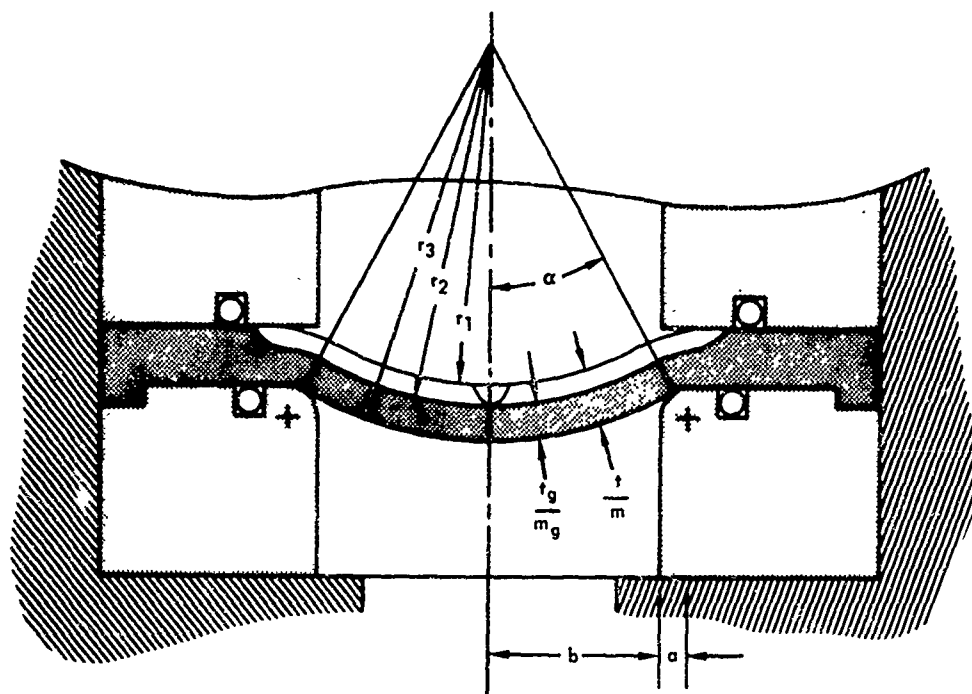
Fig. 1b **DIAPHRAGM AND HOLDER**

Fig. 2   STRAINED DIAPHRAGM

Since the same tensile force acts in the grooved portion of the diaphragm as in the wall, the stress in this part, S_g, is to the stress in the wall as the inverse ratio of their respective cross-sectional areas; that is,

$$\frac{S_g}{S} = \frac{\frac{t}{m} \Delta \ell}{\frac{t_g}{m_g} \Delta \ell} = \frac{\frac{t}{m}}{\frac{t_g}{m_g}} \tag{6}$$

where t_g is the initial thickness at the groove and m_g is its thinning coefficient.

The term stress, as used here thus far, may be referred to as the true stress as opposed to the nominal stress, the value conventionally determined in a tensile test of the material. The true stress and strain are defined[3] as:

$$\text{True stress} = \frac{F}{A} = S \tag{7}$$

where F is the load and A is the actual cross-section after straining.

$$\text{True strain} = \ln \frac{\ell}{\ell_o} = \hat{o} \tag{8}$$

where ℓ is the length after straining and ℓ_o is the original length. The nominal stress and strain are defined as:

$$\text{Nominal stress} = \frac{F}{A_o} = S' \tag{9}$$

where F is the load and A_o is the original cross-sectional area; and

$$\text{Nominal strain} = \frac{\ell - \ell_o}{\ell_o} = \epsilon \tag{10}$$

where ℓ_o and ℓ are the original and strained length respectively. From their definitions, then, the true stress is to the nominal stress as the original cross-sectional area is to the area after straining; that is,

$$\frac{S}{S'} = \frac{A_o}{A} \tag{11}$$

and the true and nominal strains have the following relationship to one another:

$$e^{\delta} = \varepsilon + 1 = \frac{\ell}{\ell_o}. \tag{12}$$

Most of the diaphragm deformation occurs above the yield condition for the material, see Fig. 3. The change in volume of a material during the first stage of plastic deformation (i.e., prior to necking) is negligible. Hence, assuming constant volume, the original volume, $V_o = A_o \ell_o$, equals the volume after straining, $V = A\ell$, so that

$$\frac{A_o}{A} = \frac{\ell}{\ell_o}. \tag{13}$$

In the case of the element of diaphragm wall previously considered, the cross-sectional area for the elemental width $\Delta\ell$ is $t\,\Delta\ell$ before straining and $t/m\,\Delta\ell$ afterwards so that

$$\frac{A_o}{A} = \frac{t\,\Delta\ell}{\dfrac{t}{m}\,\Delta\ell} = m. \tag{14}$$

The thinning coefficient, therefore, corresponds to the ratio of the initial to strained cross-sectional areas, and the following relationships between the thinning coefficient and true and nominal stress and strain are obtained:
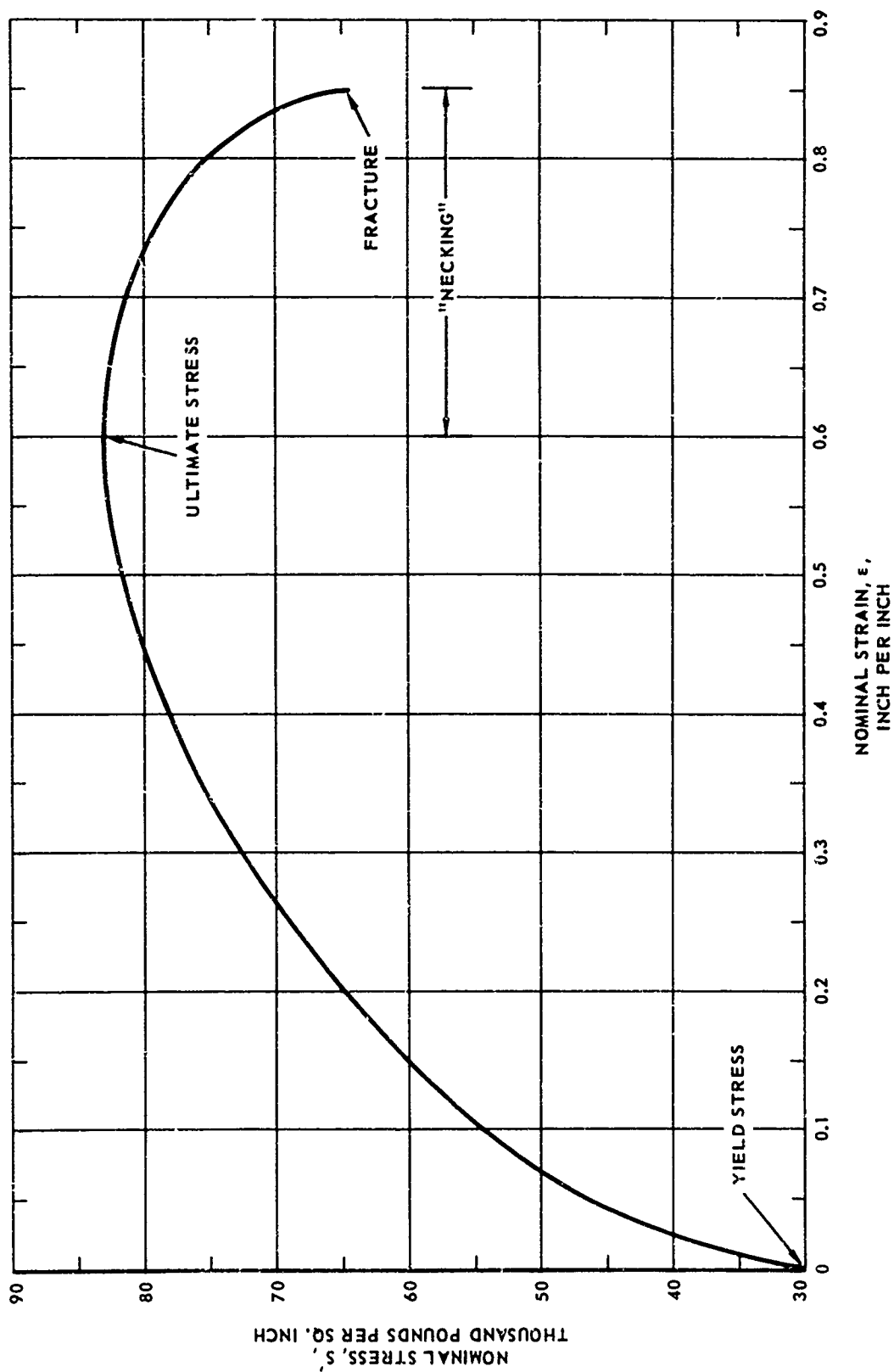
Fig. 3   STRESS—STRAIN DIAGRAM, 304 STAINLESS STEEL PLASTIC REGION

$$\frac{S}{S'} = e^{\delta} = \epsilon + 1 = m. \tag{15}$$

Equation (5) may now be written in terms of the nominal stress as

$$S' = \frac{Pr_{1*}}{2t} \cdot \tag{16}$$

Equation (6) may be written as

$$\frac{S'_g}{S'} = \frac{t}{t_g} \tag{17}$$

since the above relationships of stress, strain and thinning coefficient would also hold for the material in the grooved part of the diaphragm.

The nominal stress in the grooved portion of the diaphragm is greater than that in the rest of the wall by a factor of $\frac{t}{t_g}$ (t_g being less than t), and the diaphragm will rupture at the groove when S'_g reaches the ultimate stress for the material, S'_u. The nominal stress in the wall at the burst condition, then, is proportional to the thickness ratio; that is,

$$(S')_b = \frac{t_g}{t} S'_u. \tag{18}$$

Since stress and strain are functions of one another as determinable from a tensile test of the material, the strain and, thereby, the thinning coefficient are also functions of the thickness ratio at the burst condition.

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* This is the familiar expression for the stress in a thin-walled sphere where the stress is assumed to be evenly distributed across the wall thickness.

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Referring to Fig. 2, the length at any cross section of the part of the diaphragm in the opening is $2(a + b)$ before straining and $2(a\alpha + r_3\alpha)$ afterwards, so the nominal strain at any point in the deformed portion may be represented as

$$\varepsilon = \frac{\ell - \ell_o}{\ell_o} = \frac{2(a + r_3)\alpha - 2(a + b)}{2(a + b)} = \frac{(a + r_3)\alpha}{a + b} - 1. \tag{19}$$

Noting in Fig. 2 that

$$\frac{a + b}{a + r_3} = \sin \alpha \tag{20}$$

the strain may be written in terms of α or r_3. The strain and, thereby, the stress and thinning coefficients are all functions of r_3 for an opening of radius b with rounded edge of radius a. Thus r_3 for a given opening is also a function of the thickness ratio at the burst condition. Equation (5) may be written in terms of r_3, since $r_1 + \frac{t}{m} = r_3$, as

$$S = \frac{P}{2}\left(\frac{mr_3}{t} - 1\right) = mS'. \tag{21}$$

Therefore, for a particular opening and diaphragm material, the burst pressure may be determined for a given set of values of t and t_g since S', m and r_3 at the burst condition are all functions of $\frac{t_g}{t}$.

A typical nominal stress-strain diagram for 304 stainless steel is shown in Fig. 3. The relationship of α to the nominal strain and the thinning coefficient is presented in Fig. 4. Figure 5 shows the true
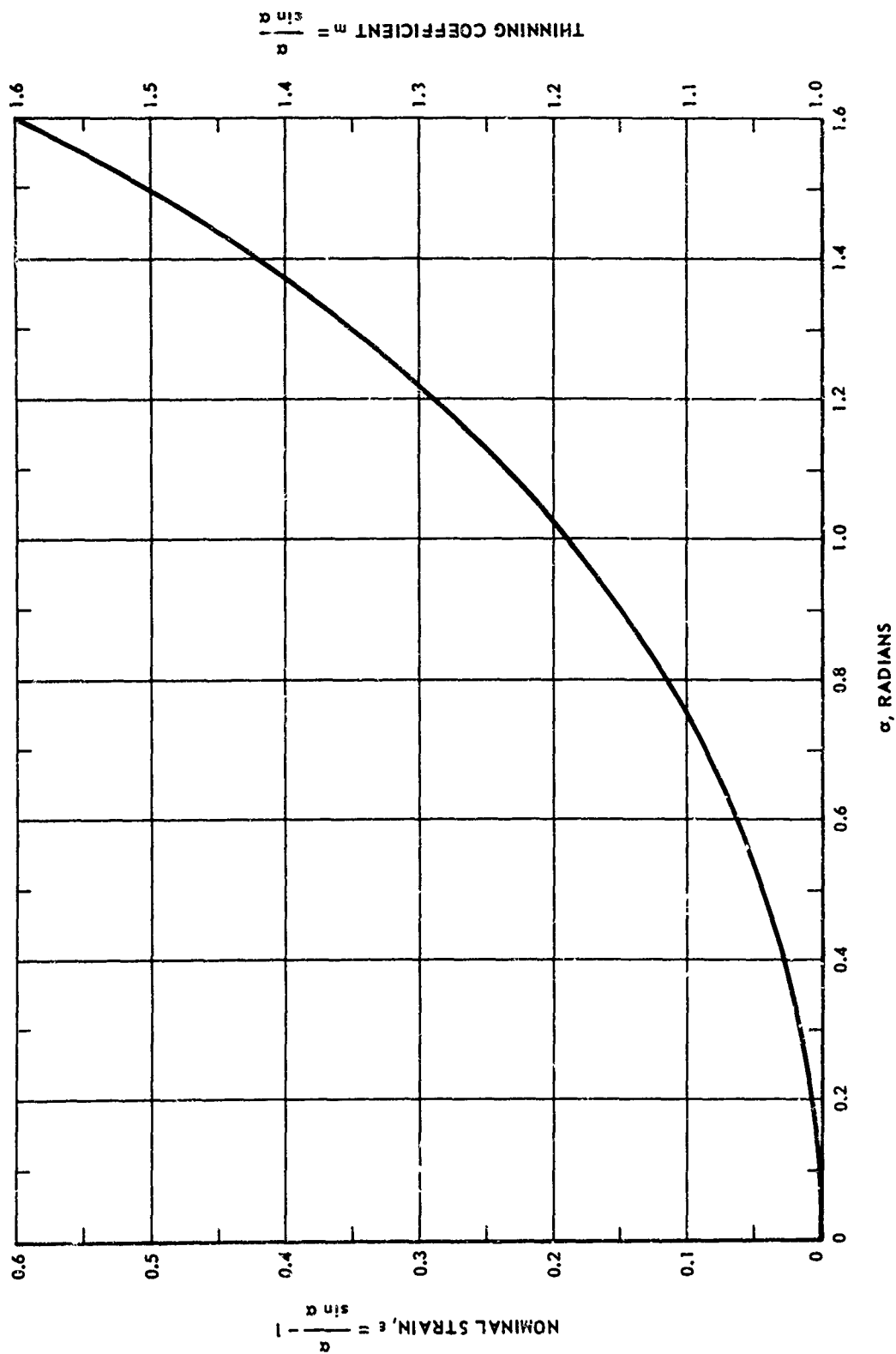
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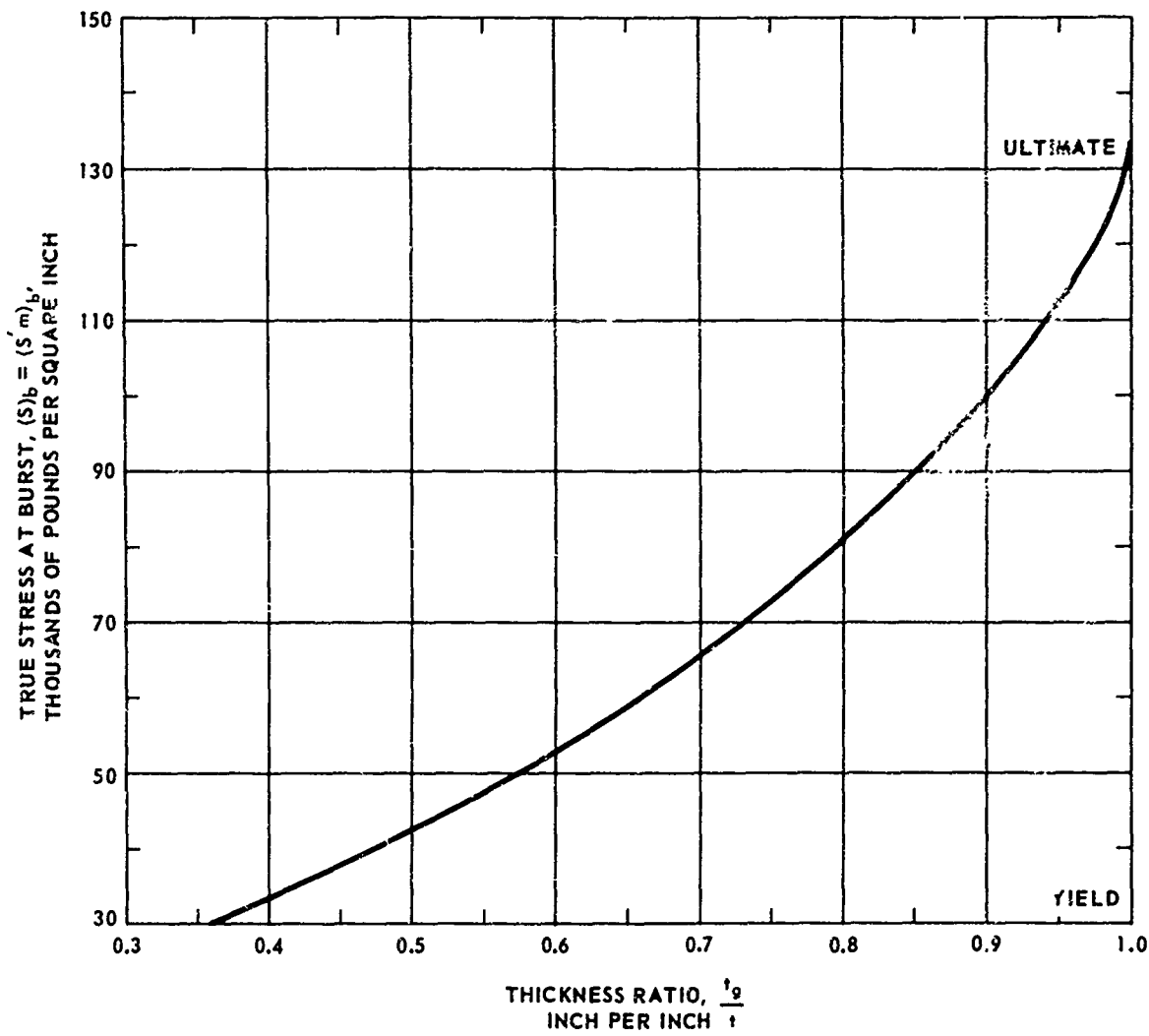Fig. 4 THINNING COEFFICIENT AND NOMINAL STRAIN VERSUS INCLUDED HALF ANGLE

Fig. 5 TRUE STRESS AT BURST VERSUS THICKNESS RATIO

stress in the diaphragm wall at the burst condition as a function of the thickness ratio; and Fig. 6 shows the relationship of thickness ratio to α at the burst condition. These curves are generally applicable to the design of 304 stainless steel diaphragms of the type shown in Fig. 1 and may be used for any given size of opening to determine the thicknesses required for a given burst pressure. For an opening of radius b having a rounded edge of radius a, α may be determined with relationship to r_3 from Equation (20) and referring to Figs. 4 and 6 a plot made of mr_3 vs $\frac{t_g}{t}$. From this and the relationship of S to $\frac{t_g}{t}$, Fig. 5, burst pressures may be computed and plotted against t for various values of $\frac{t_g}{t}$ as has been done in Fig. 7 for the gun tunnel opening.

RESULTS AND CONCLUSIONS

Average values of diaphragm burst pressure and thickness based on data from forty-seven gun tunnel tests are shown in Fig. 7 with their thickness ratios for comparison with the analytically predicted values. These experimental values are for the 304 stainless steel diaphragms initially used in the APL Hypersonic Gun Tunnel. The majority of tests were made at the 5500 and 7500 psi levels. The variations in nominal burst pressure for the diaphragms tested were on the order of $\pm$ 500 psi and the variation in thickness on the order of $\pm$ .002 inches. Petalling was satisfactory for those diaphragms having nominal burst pressure of 3500 and 5500 psi, erratic at the 7500 psi level, and generally poor for those above. This fall-off in petal behavior with increasing pressure (and
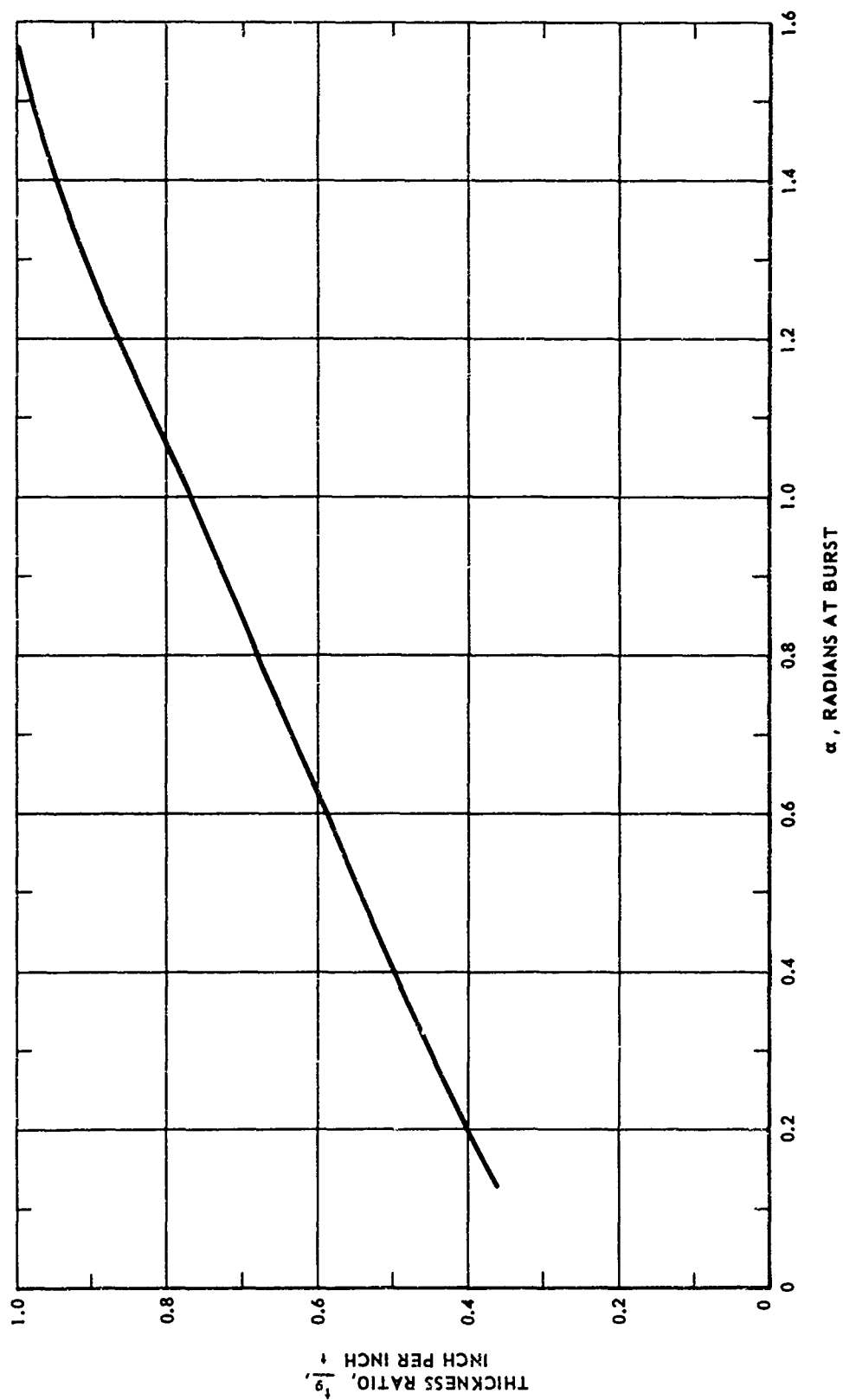
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Fig. 6 THICKNESS RATIO VERSUS INCLUDED HALF ANGLE AT BURST
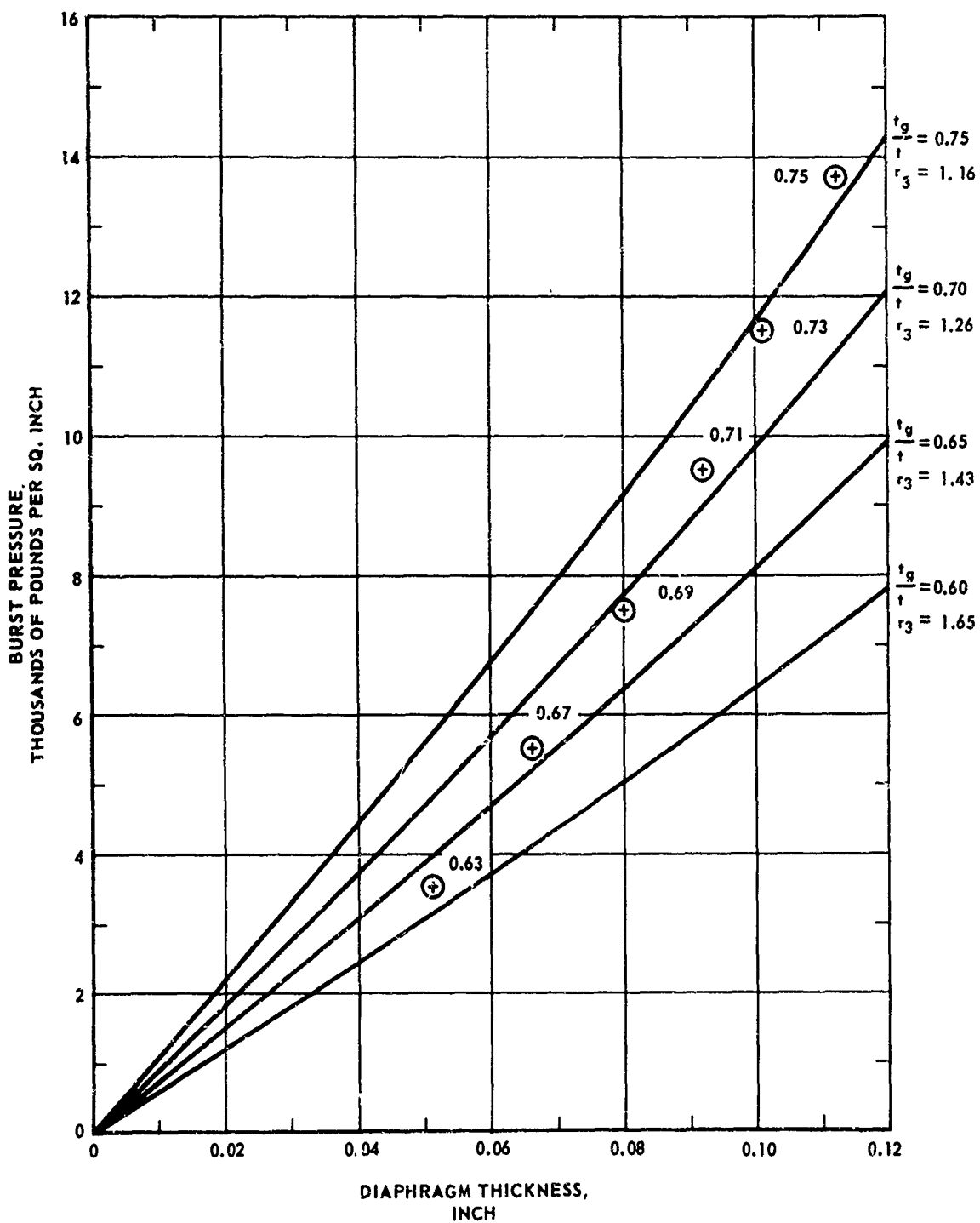
Fig. 7   BURST PRESSURE VERSUS THICKNESS FOR VARIOUS THICKNESS RATIOS

thickness) is accompanied by a decrease in the radius of diaphragm curvature
which may be seen in Fig. 7 where the outside radii corresponding to the
thickness ratio for the curves are also given. Figure 8 uses the radii
determined from the thickness ratios for the 3500 and 13,500 psi diaphragms
to compare the amount of petal projection into the opening.

Where reflected shock waves are present, the amount of diaphragm
petal curvature can be a factor in petal damage. The extent to which
pressure and thickness are involved in petalling is not immediately
evident from this study; however, an optimum value, or range of values,
of thickness ratio apparently does exist for satisfactory petalling.
This value is on the order of 0.65 for the diaphragms studied. Further
investigation of this and the other factors could prove to be very useful
in eliminating petalling problems.

The good correlation of experimental and analytical
results indicates that the method of analysis may be generally useful in
designing rupture diaphragms. It should be noted that the edge of the
diaphragm is held at its periphery so that lateral slippage is essentially
eliminated. The degree to which burst pressure may be fixed is limited
in practice by the uniformity of material and thickness that can be attained
in fabrication. The deviation in burst pressure for the diaphragms tested
is less than would be predicted analytically for the observed thickness
tolerance and is as small as can reasonably be expected in this case. The
tolerances in any given situation will depend on the opening size, material
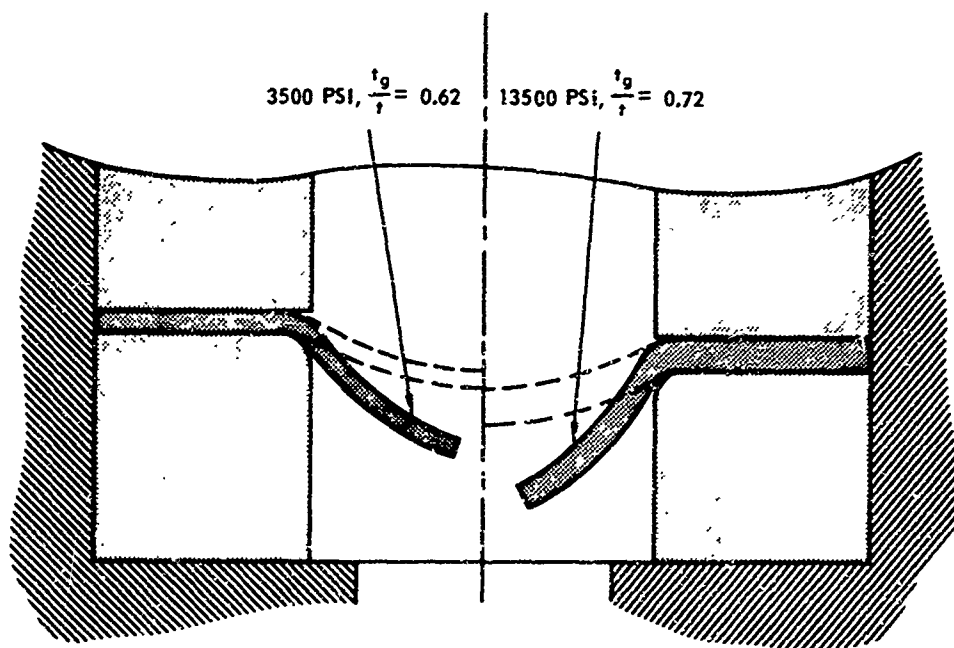and pressure involved.

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$$3500 \text{ PSI}, \frac{t_g}{t} = 0.62 \qquad 13500 \text{ PSI}, \frac{t_g}{t} = 0.72$$

Fig. 8   COMPARISON OF PETAL PROJECTION

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13. ABSTRACT

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